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COGNITIVELY GUIDED INSTRUCTION (CGI): WHAT'S THE POINT? A LOOK  
INTO CGI AND CGI'S POTENTIAL ROLE IN AN 8TH GRADE MATHEMATICS  
CLASS

by

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## STATEMENT BY THE AUTHOR

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This study examined the current research that exists on Cognitively Guided Instruction (CGI) and how it would look when implemented into an 8th grade mathematics classroom. Additional questions examined are if CGI would develop strong student understanding between mathematical concepts and their real world applications; if CGI techniques and questioning impact the interest level of mathematics students; and what type of extra resources would be needed by teachers to implement Cognitively Guided Instruction into their curriculum. Although much of the CGI research has been done on elementary students, by also examining key algebra practices the author was able to synthesize what a Cognitively Guided Instructional classroom would look like in an 8th grade setting. CGI was found to positively impact the interest level of students; with real world application as a motivator. Because students are engaged with the problems, they are able to establish connections between their own knowledge and the concepts being presented. These connections lead to stronger understanding of the mathematical procedures and produce deeper understanding.

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## Chapter 1: Introduction

### Setting

Since the passing of the No Child Left Behind (NCLB) legislation, schools and teachers have been pushed to find a methodology for teaching mathematics that will allow lower students to achieve. Teachers also strive to push the higher students to deeper levels of thought and understanding. NCLB sets the expectation that teachers will help all students reach achievement levels higher than they have reached before. Many administrators and consultants focus on what the teacher is doing in the classroom to drive instruction instead of the learner. One method that breaks this trend is Cognitively Guided Instruction (CGI). According to the Wisconsin Center for Education Research:

Elementary age students bring lots of things to school with them—besides huge backpacks stuffed with supplies. They bring ingenuity, intuitive knowledge, and mathematical insight. They sometimes amaze their teachers with innovative ways to solve problems. When mathematics teachers link their classroom instruction to students' intuitive knowledge, students can take classroom instruction a lot farther (2007, p.1).

The author currently works at a school that is classified as a 'turn-around school' by the Minnesota Department of Education (MDE). A turn-around school is identified by the State of Minnesota as persistently low achieving and in order to rapidly and dramatically increase student achievement these schools receive special funding. One foci of the school's new mission statement is that they will implement Cognitively Guided Instruction (CGI) into mathematics classes. The elementary school has implemented this into their classrooms for the past two years, the high school attempted

to infuse some of the methodologies this year, and starting next year the middle school is integrating CGI into their mathematics classes. It is because of this push by the district, the learning styles of students coming into the classroom, and professional interest in this methodology that the author is writing this paper.

### **Statement of the Problem**

With the adoption of the 2007 Minnesota K-12 Academic Standards in Mathematics (Minnesota Department of Education, 2012c) and the Minnesota Comprehensive Assessment III (MCAIII), teachers are held accountable to a more robust and rigorous level of mathematical benchmarks. Considering this and the author's district adoption of CGI in mathematics, the author intends to investigate how this would look in the 8<sup>th</sup> grade classroom. There are limited publications about CGI at the middle school level, so this paper will examine research on CGI at the elementary schools. Research on middle school algebra instruction methods will also be reviewed. It is hoped that research will help answer the following questions:

- What is Cognitively Guided Instruction (CGI)?
- Do students in a CGI classroom develop a strong understanding between mathematical concepts and their real world applications?
- How does CGI impact the interest level of students in the classroom?
- What are key components for students understanding algebra?
- Could CGI methodologies help students learn algebra?
- What type of extra resources, preparation or professional development, is needed for teachers to implement CGI into their curriculum?

## **Significance of the Problem**

The question of the importance of CGI in an eighth grade mathematics classroom is one that concerns all eighth grade mathematics teachers, while still being important to all junior and senior high school mathematics teachers. This topic has personal interest since the author works in a school identified by MDE as the one of the bottom five percent for achievement in the state of Minnesota. NCLB expects all students to have a 90% proficiency rate. Many schools are struggling to meet that goal. Dr. David Sortino had this observation:

Factors that could be affecting the test scores and not mentioned ... is that in most classrooms, teachers must teach to three and sometimes four different cognitive developmental levels at one time. The cognitive developmental levels are connected not so much to intelligence or motivation but simply to maturity or age (2011, para. 3).

Cognitively Guided Instruction has been presented as a teaching method that will reach out to those different levels. It will work with any curriculum and can be tailored to each individual teacher. Instead of focusing on direct instruction from the teacher, teachers are asked to make instructional decisions based on students' thinking. For example, teachers can take a multiplication problem and look at 1) the strategies students will use to solve it, 2) how different strategies build on each other, and 3) how those solutions relate to other problems using other operations (Loef Franke & Kazemi, 2001). While most teachers will call this common practice; Carpenter, Fennema, Peterson, and Carey (1988) stated that although teachers could distinguish between problem types and the strategies that children would use to solve them, they did not organize this

information into a coherent network that related the problem, children's solutions and problem difficulty. He went on to claim that to address this problem, CGI was designed to help teachers create maps of the development of children's mathematical thinking in specific content domains (Carpenter, Fennema, Loef Franke, Levi, and Empson, 2000). By having teachers understand the development of mathematical thinking in their students, they changed their fundamental practices and these changes were also reflected in their students' learning.

In order to help students succeed in a new world of instant gratification and visual stimulation, teachers should change the role of students to constructing mathematical knowledge rather than passively absorb it (Borko & Putnam, 1996). Through the use of CGI, teachers are able to tailor instructional practices to create a learning environment that fits their teaching style, knowledge and students. In such classrooms students are not shown how to solve problems by the teacher. Instead, the teacher encourages each child to solve problems in any way he or she can, and then a group discussion occurs between peers and the teacher as to how the problem was solved (Secada, Fennema, & Adajian, 1995).

### **Limitations**

There currently is a lack of published Cognitively Guided Instruction research on students learning algebra in the middle school, on July 1<sup>st</sup> the author did an ERIC search at the Bemidji State University library for "Cognitively Guided Instruction", "CGI" and algebra and found two results; *Teacher Questioning to Elicit Students' Mathematical Thinking in Elementary School Classrooms* by Franke et al. and *Developing Conceptions of Algebraic Reasoning in the Primary Grades, Research Report* by Carpenter & Levi.

These resources target students younger than eighth grade, so the author will examine the elementary CGI model and key components to students understanding of algebra to determine how to apply those principles to the middle school mathematics classroom. This works well with the view that teachers should be systematically and explicitly infusing algebraic concepts in elementary grades to help students avoid any misconceptions that may occur at a later grade level (Keterlin-Geller, Jungjohann, Chard, & Baker, 2007). Since Cognitively Guided Instruction focuses on the student's ability to construct solutions, overall class sizes will not be a determining factor in addressing the questions.

When observing different classrooms where Cognitively Guided Instruction was implemented, several layers of beliefs and practices were found. They were classified in the following way:

- *Level 1* teachers believe children need to be explicitly taught how to do mathematics.
- *Level 2* teachers begin to question whether children need explicit instruction in order to solve problems, and the teachers alternately provide opportunities for children to solve problems using their own strategies and show the children specific methods.
- *Level 3* teachers believe that children can solve problems without having a strategy provided for them, and they act accordingly.
- *Level 4a and 4b* teachers conceptualize instruction in terms of the thinking of the children in their classes (Carpenter, Fennema, Loef Franke, Levi, & Empson, 2000).

Since the author's classroom beliefs and practices coincide with their description of a Level 3 teacher, this paper will be written from a Cognitively Guided Instructor's view that students can solve problems without having a strategy provided for them.

### **Definitions**

**AYP-** Adequate Yearly Progress:

Adequate Yearly Progress (AYP) is a means of measuring, through standards and assessments, the achievement of No Child Left Behind's (NCLB) goal.

(Minnesota Department of Education, 2012a, para. 1).

**CGI-** Cognitively Guided Instruction:

a professional development program based on an integrated program of research on (a) the development of students' mathematical thinking; (b) instruction that influences that development; (c) teachers' knowledge and beliefs that influence their instructional practice; and (d) the way that teachers' knowledge, beliefs, and practices are influenced by their understanding of students' mathematical thinking (Carpenter et al., 2000, p. 3).

**Constructivism-**

Constructivism is a type of learning theory that explains human learning as an active attempt to construct meaning in the world around us. Constructivists believe that learning is more active and self-directed than either behaviorism or cognitive theory would postulate (Fritscher, 2008, para. 1).

**MDE-** Minnesota Department of Education:

The Minnesota Department of Education serves public school students from K through 12 and their families; young children participating in a variety of early

learning programs, including Head Start and Early Childhood Family Education; and adult learners participating in adult education programs, including GED and citizenship programs. It also serves the state's 339 school districts and more than 52,000 licensed teachers (Carlson, 2012, para. 1).

**NCLB-** No Child Left Behind: Public Law PL 107-110, the No Child Left Behind Act of 2001. “At the core of the No Child Left Behind Act were a number of measures designed to drive broad gains in student achievement and to hold states and schools more accountable for student progress” (Education Week, 2004, para. 3).

**NCTM-** National Council of Teachers of Mathematics: “The National Council of Teachers of Mathematics is a public voice of mathematics education, supporting teachers to ensure equitable mathematics learning of the highest quality for all students through vision, leadership, professional development, and research” (National Council of Teachers of Mathematics, 2012, para. 1).

**PLC-** Professional Learning Communities: “A PLC is an ongoing process used to establish a schoolwide culture that develops teacher leadership explicitly focused on building and sustaining school improvement efforts” (Center for Comprehensive School Reform and Improvement, 2012, para. 1).

**Relational Thinking-** Relational thinking depends on children being able to see and use possibilities of variation between numbers in a number sentence. Relational thinking relies on seeing possibilities of variation between numbers on either side of the equal sign, such as  $73 + 49 = 72 + 50$  (Stephens, 2012, para. 2).

**Turn Around School-**

“A School that receives the federal Title I School Improvement Grant (SIG). This grant provides funding and support to the state's identified Persistently Lowest Achieving schools in order to rapidly and dramatically increase student achievement. Minnesota SIG schools are implementing comprehensive intervention models designed to build capacity for sustainable improvement” (Minnesota Department of Education, 2012b, para. 1).

**WCER-** Wisconsin Center for Education Research: “WCER provides a dynamic environment where some of the country's leading scholars conduct basic and applied education research. The WCER portfolio includes research centers and projects that investigate a variety of topics in education” (Wisconsin Center for Education Research, 2012, para. 1).

## Chapter 2: Review of the Literature

### Defining Cognitively Guided Instruction

Cognitively Guided Instruction is based on an integrated program of research focused on the development of students' mathematical thinking; on instruction that influences that development; on teachers' knowledge and beliefs that influence their instructional practices; and on the way that teachers' knowledge, beliefs, and practices are influenced by their understanding of students' mathematical thinking (Carpenter, Fennema, Loef Franke, Levi, & Empson, 1999, p. 105).

CGI is an approach to teaching mathematics rather than a curriculum. Carpenter goes on to say even though teachers have a great deal of intuitive knowledge about their students' mathematical thinking, it was unorganized and incomplete and as such, it did not affect the teacher's decision-making about mathematic instruction in their classroom (Carpenter, Fennema, Loef Franke, Levi, and Empson, 2000). They designed CGI so the core of this approach is the practice of listening to children's mathematical thinking and using it as a basis for instruction.

As teachers revise and restructure their lessons, they create unique CGI classrooms where the teaching and learning environment are tailored according to the teacher's style, knowledge, beliefs and students (Fennema, Carpenter, & Loef Franke, 1992). These classrooms are built on the belief that each student's thinking is important and respected by both the teacher and their peers. Students realize their thinking is important to solve the problem and they become flexible in their approaches to problem solving.

They are perceived by the teacher to be in charge of their learning, using mathematical strategies already known to solve meaningful contextualized problems (Fennema et al., 1992).

### **Cognitively Guided Instruction: In the Elementary Classroom**

In CGI classrooms, teachers focus on the student's thinking rather than specific procedures or curriculum materials (Wisconsin Center for Education Research [WCER], 2007). Keeping this in mind, much of the class time is focused on students solving non-routine word problems. Word or story problems are a key aspect of a CGI classroom, they are a powerful tool to engage students in mathematics and many students enjoy finding solutions to the situations presented (Jacobs & Ambrose, 2008). Story problems allow the teacher to bring real world connections to the mathematics being presented to the class. Another strength they provide is that children are able to intuitively solve word problems because they are able to model the relations and actions presented by the problem (Carpenter et al., 2000). Teachers stimulate student thought and interaction with a problem related to an earlier situation presented in the class, a unit or theme from another core class, or something going on in either the students' or teacher's lives (Fennema et al., 1992). This creates a connection between the curriculum and their experiences, giving them the best opportunity to learn and succeed. The National Council of Teachers of Mathematics confirms this belief in their statement about the connections standard for grades 3-5:

Students in grades 3-5 study a considerable amount of new mathematical content, and their ability to understand and manage these new ideas will rest, in part, on how well the ideas are connected. Connecting mathematical ideas includes

linking new ideas to related ideas considered previously. These connections help students see mathematics as a unified body of knowledge rather than as a set of complex and disjoint concepts, procedures, and processes (2000, p. 200).

Using story and word problems to create connections to children's thinking, learning and mathematical concepts fills the needs of multiple types of learning styles. Consider the style-based learning dispositions as described by Strong, Thomas, Perini and Silver:

- **Mastery:** Students in this category tend to work step-by-step.
- **Understanding:** Students in this category tend to search for patterns, categories, and reasons.
- **Interpersonal:** Students in this category tend to learn through conversation and personal relationship and association.
- **Self-Expressive:** Students in this category tend to visualize and create images and pursue multiple strategies (2004).

Mastery students are students who are grounded in computational skills. These type of students need individual time to work through and solve problems themselves. At this time, the teacher is moving around the classroom and providing individual questions and guidance so students come to a proposed solution to the problem (Franke et al., 2009). If a student is unable to start working on the problem, the teacher asks the student to explain the problem. According to Jacobs and Ambrose (2008), by having them describe the problem in their own words, the teacher can pinpoint what they do and do not understand. Teachers then clarify any individual misunderstanding about the problem and allow students to continue to pursue solutions. Although teachers are effortlessly

able to ask initial questions to start students' mathematical thinking, they struggle with how to follow up once a student presents their ideas (Franke et al., 2009). It is more difficult to follow up on student explanation, and teachers need to support student thinking as they attempt to construct connections between their personal strategy and one presented by a fellow classmate. This is where student talk can lead to increased student mathematical knowledge and understanding. It allows the teacher to assess students' mathematical thinking, providing teachers an instructional direction for follow-up questions. Student talk also makes it possible for children to gauge each other's strategy and comprehension (Franke et al., 2009).

A group discussion on how the problem was solved occurs between the students, their peers and the teacher. During class discussions the teacher must make students' mathematical thinking explicit. Students share their ideas with the class; describing what they did, explaining what steps they took to solve the problem, and justifying why their reasoning is valid (Carpenter & Levi, 2000). The teacher then finds another student or group who used a solution strategy different from the ones previously discussed (or a similar strategy approached in a different way), and goes through the same process for each new strategy presented. Children need the opportunity to solve, explore mathematical connections, and examine multiple strategies that occur from the same problem (Jacobs & Ambrose, 2008). The teacher is expected to ask the children to compare and contrast different solutions for a problem. After the students explain and justify their strategies, the responsibility of the validity of a strategy falls not only upon the teacher but the students as well (WCER, 2007). These types of discussions are also encouraged by the National Council of Teachers of Mathematics (NCTM):

In classroom discussions, students should become the audience for one another's comments. This involves speaking to one another in order to convince or question peers. The discourse should not be a goal in itself but rather should be focused on making sense of mathematical ideas and using them effectively in modeling and solving problems. The value of mathematical discussions is determined by whether the students are learning as they participate in them (2000, p. 194).

It is through these discussions that the teacher meets the needs of the last three learning types, those of the understanding, interpersonal and self-expressive learners. Through the discussion process the teacher and students will examine multiple solutions to try to find patterns, use personal experiences and conversations to justify the validity of those solutions, and determine which solutions are valid or if there are solutions that the class may have missed.

After the class has discussed multiple, correct solution paths to a problem, it is time for them to apply their newly formed knowledge on practice exercises. A drill method is appropriate, but only after extensive conceptual development has taken place (Kloosterman & Gainey, 1993). Kloosterman and Gainey (1993) state that the reliance on excessive drill causes students to view mathematics as unrelated fact sets, but that the "mathematical concepts are best remembered if they are taught in relation to already stored information. Thus teachers should explain new information in the terms of knowledge students already possess" (p. 8). The practice assignment should contain both similar questions to the discussion problem as well as problems that encourage students to apply what they learned to new situations. Teachers should encourage students to

connect their own methods and prior learning to new concepts. That way when students are confronted with a non-routine problem, they will have the confidence to approach the problem in multiple ways to find a solution. They will make connections between these new problems and the ones they know how to solve (Kloosterman & Gainey, 1993).

Assessments should also support this methodology. There should be space allocated on each assessment that will address the different learning style of the students. An outline of such an assessment would be similar to this: the beginning of the assessment would address the standard. Below that, a four-part grid would follow, giving space for the four different ways that students learn. One would be a computational space, a second space where the student explains how the operation works, a third for applying the operation to real-life situations, and finally a space that solves a non-routine problem using that operation (Strong, Thomas, Perini, & Silver, 2004).

### **Cognitively Guided Instruction: Contextualized Problems**

“Contextualized instruction is based on developing new skills, knowledge, abilities, and attitudes in students by presenting new subject matter in meaningful and relevant contexts: contexts of previous experience, real-life, or the workplace. New skills are then applied in these relatable contexts. Key words that describe this method are applied, relatable, relevant, and authentic” (Carrigan, 2008, p. 1).

Using contextualized problems in mathematics is a preventative measure for errors or misconceptions. When students struggle in mathematics, teachers give students a rule to follow as a quick fix. This may appear to solve the immediate problem but it also interferes with students’ development of the mathematical understanding of the problem type (Behrend, 2001). Behrend goes on to explain when students are given rules to

perform, but they do not understand why the rules work, they apply them indiscriminately. They blindly accept the rules, and begin to have no expectation that mathematics should make sense. Carpenter and Levi (2003) suggest that giving students instructional contexts where their implicit knowledge can be made explicit is the goal, and contextualized problems grant access to that goal. Students can take a problem that conveys experiences they are familiar with and create connections and relations to the underlying mathematic concepts embedded in the problem.

The use of contextualized problems seems like a trivial concept, but when you consider what strategies young children, and even adults, employ to solve problems; contextualized problems enhance that process. Baek and Flores (2005) state: “[i]nitially young children employ direct-modeling strategies using physical objects to represent quantities, actions and relationships in a problem” (p. 54). Contextualized problems give students a clear, concise direction for this modeling and present a starting point for the solution process. This is important at any level you are teaching; as adults tend to use the same strategies when presented with non-routine problems (Baek & Flores, 2005). Let the problems have meaning, problems the students can see as being important or have some real world application, problems that they may have to solve themselves sometime. Most textbooks have these problems in their practice set, teachers need to make them the focus of the lesson and place less emphasis on drill and routine problems, since they really have very little importance outside of the academic setting (Thorpe, 1989).

### **Cognitively Guided Instruction: Linking Concepts to Real World Applications**

“By the time algebra is introduced in middle school, many students view mathematical principles as subjective and arbitrary and rely on memorization in lieu of

conceptual understanding” (Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005, p. 67). This is one of the foci CGI attempts to correct, student’s ability to create connections with mathematical concepts by using the prior skills and knowledge they have obtained. The use of contextualized problems helps teachers foster that connection and learning process. Students actively construct their own knowledge to concepts that are presented. This process is known as constructivism, and it suggests that rather than simply accepting new information without question, that learners process and interpret the information they see and hear. Students remember the new information according to skills and concepts they already know (Kloosterman & Gainey, 1993). Kloosterman and Gainey (1993) go on to say many middle school students have not acquired formal operation knowledge, but instead can understand mathematical concepts through concrete representation. Johanning (2004) confirms this theory by noting that by giving a student adequate time to explore the problem context (and in some cases some interventional help), they were able to make sense and use relationships to solve problems. Children often have multiple ways of thinking about and solving mathematics that differ from adult perspectives. Teachers must instruct in such a way that it builds on students’ ways of thinking so that the classroom becomes a rich instructional environment that produces gains in student achievement (Jacobs & Philipp, 2010).

“We found CGI teachers placed greater emphasis on problem solving and less on computational skills, expected more multiple-solution strategies rather than a single method, listened to their children more, and knew more about their children’s thinking” (Carpenter et al., 2000, p. 4). Adapting curriculum to their student’s experiences and lives, these teachers are creating an atmosphere in their classroom where students expect

to be challenged yet feel safe and approach the problem in any method they deem appropriate. When new concepts are introduced, if students are applying prior knowledge to the situation, they are reviewing and reinforcing the knowledge being taught. This allows them to see the overall picture of how the concept fits into mathematics as a whole rather than a separate procedure (Shuhua, Klum, & Zhonghe, 2004). This overarching view of the interconnectivity of mathematics is what teachers want to support.

“How often do we hear curriculum developers and teachers say, ‘I want to make mathematics simple for students?’” (Chen, 2012, p. 465). This is a result of teachers and developers straying from teaching in a holistic manner and simply presenting students with a problem solving “step-by-step” manner students can easily follow. By simplifying the curriculum teachers are depriving students the opportunity to access intrinsic intuition that is inherent in human experiences and knowledge, which can actually impede learning (Chen, 2012). For many teachers, their textbook curriculum has become their district’s pre-described academic standards. Their instructional pacing is a race to cover the standards prescribed by their state assessment, and the only goal of teaching is to raise the student scores on that test (Tomlinson, 2000). When the overall goal is teaching to standards, students lose track of making connections between mathematical concepts and their applications to situations outside of the classroom.

Jacobs and Philipp (2008) instead suggest teachers request links between strategies and their symbolic mathematical notation. Children will see the connection to the mathematics done on their paper and the story or word problem presented to them in class. This is practical experience for students since most employers often expect their

employees to translate work-related problems into symbolic notation. Examples of this are calculating discounts for merchandise, tracking sales of merchandise in order to maintain a stock of items, to operating technology-based machinery and equipment. In science and technology careers, mathematical competence is expected and required to solve complex problems; such as chemical equations, computer simulation and drug interaction (Keterlin-Geller et al., 2007). Students who are able to connect mathematical procedures with underlying concepts are more likely to correctly apply them to a new situation rather than in an inappropriate manner (Kloosterman & Gainey, 1993).

### **Cognitively Guided Instruction: Student Interest and Participation**

“One of the things that was striking about the classes in which we worked was that the students were engaged in sense making. They thought that mathematics should make sense and that they could make sense of it. Students persisted for extended periods of time working on a problem; because they thought they should be able to figure it out” (Carpenter & Levi, 2003, para. 12).

This is commonplace in CGI classrooms; much of the class period entails students working on a problem whose theme connects with the students’ experiences. When elaboration makes a problem more meaningful, students are more likely to avoid prescribed problem-solving approaches and instead work to make sense of a problem (Jacobs & Ambrose, 2008). Students begin by constructing generalizations from their conceptions of meaningful situations and to derive their formalizations from conceptual activities based in those situations (Kaput, 2000).

After the teacher presents the class with a problem, the student’s reactions were brainstormed. They are then asked to journal about the problem and their suggested

solution. Students then present their ideas and solutions to the class, where they are discussed and a classroom consensus is taken on its validity (Brizuela & Schliemann, 2004). Students are actively engaged not only presenting and processing their own thoughts on the problem, but in analyzing their peer's approaches as well. This environment in the classroom allows students to realize that all problems are not solved easily or in a short manner. They accept that it is fine to be stuck on a problem at any point and that learning from their mistakes is a powerful part of their education (Chen, 2012). This process results in student ownership of their own learning, which in turn produces increased student interest and participation.

Our job is not to make students believe that math is easy and fun to learn. We need to help students understand that enjoyment in mathematics resides not only in fun games but also in making sense of mathematics and seeing connections (Chen, 2012, p. 471).

At first students may seem disinterested and disengaged from challenging problems, but in reality they enjoy solving problems with which they can connect contextually. Students who do not like mathematics do not see themselves as proficient in mathematics. They see little to no value of mathematical knowledge outside of school and will tend to take the less rigorous courses offered. They become more involved in mathematics that they view as having uses in their everyday life; by emphasizing real-world problems we can deepen students' understanding of the value and uses of mathematics (Hart & Walker, 1993). They will recognize that learning mathematics involves a positive struggle with the mathematical concepts and their previous learning, which can be done with challenging and complex investigations (Chen, 2012).

A Cognitively Guided Instruction teacher's goal is not to tell children the most efficient strategy, but to ask questions that will allow students to reflect on their previous experiences and consider improvements to those plans (Jacobs & Phillip, 2004). One of the best ways to have students consider these connections is to present the problems in a concrete context. Although students and teachers normally view manipulatives as play time, they are an excellent medium to allow students to make connections between their past experiences and the mathematical content they are modeling (Kloosterman & Gainey, 1993). The use of these concrete materials, models and pictures helps students explore, visualize, and connect mathematical concepts. This sparks student interest and participation, learning by doing encourages the children to acquire knowledge through creativity, inquisition and critical thinking (Shuhua et al., 2004).

Another method to increase student involvement is by differentiating the instructional presentation in class. Such strategies include, but are not limited to:

- **Rotate Strategy Focus:** over the course of a unit or theme, rotate the primary focus of your classroom between the four types of learners; mastery, understanding, interpersonal and self-expressive.
- **Flexible Grouping:** group students in varying ways, mixing between style-alike and style-diverse grouping promotes a positive discourse for discussions and analysis.
- **Personalize Learning:** when your students need an extra challenge or are struggling with a concept, switch the strategy to their personal learning style. Presenting problems in their learning type will promote understanding, interest, and participation (Strong et al., 2004).

Students will see classroom learning being driven by their own inquiry. Once they perceive mathematical knowledge as their own, they will come to believe they can construct new knowledge and learning by their own investigations and activity (Loef Franke & Kazemi, 2001).

### **Algebra: Key Components for Understanding**

Knuth, Alibali, McNeil, Weinberg, and Stephens (2005) stated that “algebra has been called the study of the 24<sup>th</sup> letter of the alphabet” (p. 69). For many students, algebra is solving for X. What X represents or what relationships are used to solve for X are overshadowed by the need to find the answer for X. Johanning (2004) described school algebra as providing a limited perspective, focusing on describing and calculating activities. In these activities students are doing algebra through creating and manipulating equations, not using algebra or implementing activities that give meaning to algebraic concepts. Kaput (2000) confirms this view by stating “the traditional image of algebra, based in more than a century of school algebra, is one of simplifying algebraic expressions, solving equations, learning the rules for manipulating symbols- the algebra that almost everyone, it seems, loves to hate” (p. 2).

The NCTM Principles and Standards for School Mathematics document states “students in the middle grades should learn algebra both as a set of concepts and competencies tied to the representation of quantitative relationships and as a style of mathematical thinking for formalizing patterns, functions, and generalizations” (NCTM, 2000, p. 223). In order to do this, algebra must be introduced and developed in the elementary grades. The overall goal is not to push traditional algebra methods and curriculum into the elementary grades, but to have students develop understanding that

the equal sign is not an operation but a relational symbol and how to think mathematically about those relations (Carpenter, Fennema, Loef Franke, Levi, & Empson, 2005). Carraher, Schliemann, Brizuela, and Earnest (2006) suggest functions also should be introduced early in the mathematics curriculum, since it will help facilitate the integration of algebra in existing curriculums. The best way to support functions at this level is to teach addition, subtraction, multiplication and division as a functional relation; enforcing the concept that a function is an operation. These types of problems can be solved using a wide variety of tools: symbolic notation, number lines, function tables, and graphs. These are powerful tools for students to use to understand functions and express functional relationships across various problem contexts. Thus the concept of adding three can be presented not only as number facts to memorize, but through various representations. One representation can be through a standard function, such as  $f(x) = x + 3$ , or a mapping notation, such as  $x \rightarrow x + 3$ . Students will understand that arithmetic operations are both number fact families as well as general statements where they find a specific solution (Carraher, Schliemann, Brizuela, & Earnest, 2006).

Carpenter (2003) believes elementary school students can learn to adapt their thinking about mathematics so it becomes more algebraic. They can learn to generalize arithmetic and to use language or symbols to express those generalizations. Students are presented problems with real world applications: “whenever possible, the teaching and learning of algebra can and should be integrated with other topics in the curriculum” (NCTM, 2000, p. 223). Teachers are not merely connecting algebra to topics or experiences that students will encounter in everyday life; they are allowing students the freedom to find a solution unique to them. Honoring this freedom is critical so strategies

they understand are constructed rather than parroted strategies presented to them by a teacher (Jacobs & Ambrose, 2008).

The point is not to teach students to use particular strategies but to think about how having students articulate their thinking and share it with others provides opportunities to develop student's ability to think algebraically while they work with approaches that are sensible to them (Johanning, 2004, p. 387).

Now that the teacher has the students at a place of understanding derived from experiences they can relate to and strategies they developed, they can push them to generalize the algebraic concept over different scenarios. Kaput (2000) describes generalization as deliberately extending the range or scope beyond the provided problem and achieving a level where the focus is no longer on the situations themselves, but rather on the patterns, procedures, structures or relations provided by the problem. Carraher et al. (2006) support this view by stating that students started instantiating unknowns to particular values when they are first introduced to algebra. After further instruction and practice, they started to use number line representations and algebraic notations to describe the events of the problems they were presented. Kauput (2000) summarizes this concept by stating "mathematics thinking ultimately arises from experience and only becomes mathematical upon appropriate activity and processing" (p. 6). Once students are able to connect and understand the problem with which they are presented, use algebraic expressions to describe that problem, and are able to modify or adapt their original expression to fit a different scenario, they are learning with understanding. Understanding develops over time, and algebraic thinking and understanding develops over multiple grades (Carpenter & Levi, 2000).

As all of this implies, considerable reform of traditional methods are required to develop algebraic thinking. Suggested elementary and middle school level adjustments include (but are not limited to):

- A focus on relations and not merely on the calculation of a numerical answer.
- A focus on operations as well as their inverses, and on the related idea of doing/undoing.
- A focus on both representing and solving a problem rather than on merely a solution.
- A focus of both numbers and letters, rather than on numbers alone.
- A refocusing on the meaning of the equal sign, not as an operation but as a relation between two quantities (Kieran, 2004).

Carraher, Schliemann, and Brizuela (2001) state that algebraic reasoning may appear to be constrained by a student's level of cognitive development, that algebraic reasoning and concepts require a cognitive maturity most elementary and middle school students do not yet possess. They go on to state that it is not because of the student's developmental constraints, but rather to curriculum where it introduces these algebraic concepts at too late a stage for students and conflicts with their intuitions and beliefs about arithmetic. This is supported by the view of Keterlin-Geller, Jungjohann, Chard, and Baker (2007): "by the time algebra is introduced in the middle school, many students view mathematical principles as subjective and arbitrary and rely on memorization in lieu of conceptual understanding" (p. 67). Understanding mathematics for many students is remembering which set of rules to apply to which set of mathematical symbols.

Unfortunately, algebraic understanding comes from connecting the student's knowledge to procedures and concepts (Kaput, 2000).

One starting point for teachers is to strengthen student's understanding of relational thinking. Empson and Levi (2011) found that adults as well as children use relational thinking intuitively. This type of thinking affects how we mentally manipulate not only multi-digit numbers, but fractions as well. Empson and Levi (2011) found students who understand numbers relationally are able to use mathematical relationships to solve problems. Encouraging students to work in this context allows students to connect their own understanding algebraic properties; such as the distributive, associative, commutative, identity properties as well as inverse relationships. These then lead into the properties of equality, which adults classify as the rules to solve algebraic equations.

Another starting point to promote this understanding with students who continue to struggle with a concept is to present them with true-false sentences. Molina and Ambrose (2006) conducted a study with eighteen third grade students over five sessions. During the first session, students were presented a written assessment with open number sentences to determine their understanding of the use of the equals sign. No student gave more than one correct answer to six different problems. Students were then presented with true-false sentences in the second session to determine how students' conceptions of the equals sign evolves when considering and discussing varied true-false sentences. Two-thirds of the class continued to have misconceptions about the equals sign. In the third session, Molina and Ambrose (2006) discussed various true-false sentences to promote relational thinking. Twelve of the eighteen students were able to solve five of

the six assessment questions. The fourth session focused on the question of once students correctly interpreted the equals sign, do they use relational thinking to evaluate true-false number sentences? Only seven of the eighteen students were able to display relational thinking. The fifth and final session the students were once again presented with open number sentences to measure if they retained their new interpretation of the equals sign. Twelve of the fifteen students assessed were able to correctly solve five out of seven problems. Molina and Ambrose (2006) found that students' conceptions evolved about the equals sign, but that they were only partially successful in initiating relational thinking. The biggest challenge they faced was the transition for students away from computation of the sentence to evaluating the whole sentence. "A few students went back and forth between their original conception and their newer conception, suggesting that developing a robust understanding of the equals sign can take considerable time" (Molina & Ambrose, 2006, p. 117).

This study showed the importance of using true-false number sentences with students. Number sentences should be used to promote relational thinking, and students can solve them by focusing on the relationships between the numbers or properties of the equation instead of performing all the computations presented by the sentence. As students construct strategies for solving true-false problems, they are using fundamental properties of addition, subtraction, multiplication, division, and the relations among the operations (Carpenter & Levi, 2000). Once students are able to solve the equations presented to them, asking the students to write their own will allow students to assimilate the new information and consolidate their conceptions. This is due to the fact they have to use the sign or property to create their own equation instead of evaluating or solving

someone else's (Molina & Ambrose, 2006). Another follow up to this would be to present the students with open sentences- often called 'missing addend' problems. These problems are usually their first experience with equation manipulation and are divorced from contextualized problems which carry semantic information about the operations (Kieran, 1989). They allow teachers to gain insight on student's understanding of the operations, and what interventions are needed. After students demonstrate and understanding of the problem, an assignment of creating their own story problem based on the open sentence is appropriate and encouraged.

Teachers should monitor their curriculum and teaching to promote an atmosphere where students are learning with understanding. Kieran (1989) points out:

the emphasis towards 'finding the right answer' in the curriculum allows children to get by with an informal, intuitive procedure set. However, in algebra, they are required to recognize and use the structure that they have been able to avoid in arithmetic (p. 39).

These aspects of algebra, recognition and structure use, are ones that appear to never really become sorted out with students throughout their entire high school career.

### **Cognitively Guided Instruction: Effectiveness with Algebra Instruction**

While there have been studies on Cognitively Guided Instruction's implementation with algebraic concepts, they are limited to the elementary grade levels, kindergarten through sixth grade. Carpenter, Fennema, Loef Franke, Levi, and Empson (2005) stated it is not the goal to push traditional high school algebra concepts into the elementary grades, but there are algebraic concepts that can be introduced in the earlier grades. Some of the skills Carpenter suggests that elementary grades should build on are:

adapting arithmetic thinking into a more algebraic nature, understanding that the equal sign does not mean an operation but rather a relationship, generating discussions using teacher selected true-false statements in instructional contexts appropriate for students, and learning to generalize and expressing those generalizations accurately using natural language and mathematical symbols (Carpenter, Loef Franke, and Levi, 2003).

Brizuela and Schliemann (2004) conducted a study with seventy students in grades 2 to 4. Each semester, from the second semester in 2<sup>nd</sup> grade to the last semester of their 4<sup>th</sup> grade, they implemented six to eight algebra activities, each lasting about 90 minutes. The activities related to operations, fractions, ratios, proportion, and negative numbers. Brizuela and Schliemann wished to examine how students worked with variables, functions, positive and negative numbers, algebraic notation, function tables, graphs, and equations. The last six activities in 4<sup>th</sup> grade focused on algebraic notation and equations. One-third of the children represented the problem using algebraic notation, and over one third of them included a variable for at least one of the unknowns in the problem.

Carraher et al., (2006) also pointed out that third grade students are able to engage in algebraic reasoning and work with function tables, given the proper challenges and contexts. He goes on to say that symbolic notation, number lines, function tables and graphs are powerful tools that students can use to understand and express functional relationships. By using number lines and variable number lines they are able to successfully implement the use of variables and functional covariation to solve problems using additive relationships.

All of these elements are readily implemented using contextualized problems, questioning techniques, and classroom discussion methods provided by the Cognitively Guided Instructional framework to classroom instruction. According to Carpenter, Loeff Franke and Levi (2003), through the processes of mathematical reasoning, existing patterns and ideas are recalled and developed into new ones. This new generation of ideas, the representing and investigation of them is the focus of elementary arithmetic that Cognitively Guided Instruction provides. Although the context and numbers used will be different, these underlying themes are appropriate for students ranging from primary to middle school.

Some of the most important skills to listen to children's thinking and use it to guide instruction are:

- Posing problems for children to solve using their own strategies.
- Choosing or writing problems that elicit a variety of valid strategies and insights.
- Adjusting problem difficulty and number choices in developmentally appropriate ways.
- Asking probing questions to clarify and extend children's thinking.
- Conducting discussions of students' strategies so that students can make new mathematical connections.
- Identifying the important mathematics in children's thinking (Empson & Levi, 2011, p. 227).

They go on to say the most significant challenge Cognitively Guided Instructors have is to listen with the intent to hear what students have to say about the problem and

their thinking about that problem, without imposing their knowledge or providing hints to the solution to the problem. It is by those discussions with struggling students that teachers are able to assess what student know and are learning and can thereby adjust instruction to meet those needs.

### **Cognitively Guided Instruction: Resources and Professional Development**

“Teachers experience a vast range of activities and interactions that may increase their knowledge and skills and improve their teaching practice, as well as contribute to their personal, social and emotional growth as teachers. These experiences can range from formal, structured, topic-specific seminars given on in-service days, to everyday, informal ‘hallway’ discussions with other teachers about instruction techniques, embedded in teachers’ everyday work lives”

(Desimone, 2009, p. 182.)

This view is supported by Borko (2004), adding learning in the classroom, with school communities (PLC), or counseling a troubled child as further elements that contribute to teacher’s professional development and growth.

Borko (2004), goes on to state that professional development given by and continued with Cognitively Guided Instruction can help teachers construct understandings on how children’s ideas about mathematics develop; and how to promote connections between those ideas and the big ideas presented in the curriculum. Teachers received a four-week summer workshop which provided teachers with strategies that students use to solve problems, the types of problems students can find difficult, and differentiation for those problems. Borko (2004) found that Cognitively Guided Instruction professional development provided an extensive focus in instructional

practices, which teachers brought into their classroom. Those teachers also focused more on problem solving and fostered class discussions on strategies used in those problems.

Desimone (2009), identified a set of critical features essential to any professional development program. They are:

- Content focus: activities should connect subject matter with how students learn that content.
- Active learning: professional development should include observing expert teachers, participating as a learner, or being observed. Interactive feedback and discussion should also be included.
- Coherence: programs should have consistency between what the teachers learn and their knowledge and beliefs. This consistency should also extend to the school, district and state.
- Duration: development promoting instructional change should comprise of two components; a long span of time for training and the number of hours of contact time within the training.
- Collective participation: teachers from the same school, grade or subject matter should participate in the program to promote interaction and discourse- strengthening teacher learning.

These features focus both on the individual teacher as well as their community, building a healthy background for teacher learning and providing support in its implementation into the classroom.

### **Chapter 3: Discussion**

#### **What does Cognitively Guided Instruction Look like When Implemented in the Classroom?**

According to Carpenter, Fennema, Loef Franke, Levi, and Empson (1999), a classroom that successfully implements Cognitively Guided Instruction contains a teacher who instructs based on students' knowledge. They discuss mathematical concepts and listen to their students' mathematical thinking, using it as a basis for instruction. The concept that students are able to intuitively solve problems is the backbone upon which this methodology was founded. Many researchers agree that the key aspect of a Cognitively Guided Instruction classroom is the use of word or story problems: Carpenter et al. (1999), Jacobs and Ambrose (2008), Kloosterman and Gainey (1993). Fennema, Carpenter, and Loef Franke (1992) state that these problems must relate to an earlier situation from class, a unit or theme from current core classes, or real life experiences in either the students' or teacher's lives. This will create a personal connection between the concept and student, a concept the National Council of Teachers in Mathematics (2000) identified as critical to student learning and success.

Carpenter et al. (1999) give an overview of the general flow of the lesson. Students are first presented with a story problem. They are then allowed a certain amount of time to work on the problem individually. After that, the teacher engages the class in a discussion of the problem and its possible solutions. During this time, students are called upon to explain their solution. If there is more than one solution path, each is presented to the class and discussed for correctness, similarities and differences from the previous (Carpenter & Levi, 2000). This encourages students to look for patterns. The use of

classroom discussions is another focal point for the National Council of Teachers of Mathematics (2000), teachers should help students make sense of the problem and the mathematical concepts needed to successfully solve the problem.

The use of word problems in a Cognitively Guided Instructional setting addresses multiple learning styles. Strong et al. (2004) categorized these styles as mastery, understanding, interpersonal and self-expressive. Interpersonal learning types are immediately addressed through these problems, and Jacobs and Ambrose (2008) state that through discussion teachers can pinpoint student learning and deficiencies. Individual work at the onset of the problem fulfills the need of mastery students, and group discussion of the solutions allows understanding students to engage and connect to the lesson. Self-expressive students can shine throughout the class time; through pictures and diagrams depicting the problem to sharing and analyzing multiple solution strategies. Jacobs and Ambrose (2008) also agree that students need these discussions of multiple representations and solutions of the problem.

Once the class has worked through the problem and its correct solutions, the teacher explicitly connects student's concepts to formal mathematical concepts presented in the curriculum (Franke et al., 2009). Kloosterman and Gainey (1993) agree that the formal component of the lesson should only come after students form their own understanding of the problem, and that drill and practice should only be implemented after students have a conceptual development of the problem. This will strengthen students' problem-solving proficiency, and they will have the confidence to approach non-routine problems in multiple ways.

Assessments in a Cognitively Guided Classroom can occur in a number of different ways. Teachers can gauge individual learning through the independent work portion of the class. They will gather information on class knowledge through the discussion of solution paths and their validity. Strong et al. (2004) also give a template for formal assessments, containing a balance of the four learning styles: computational problems, explanation of the operation, application of the operation and finally a non-routine example for which the operation must be applied.

### **Impact of Cognitively Guided Instruction on Students and the Classroom**

Carpenter et al. (2005) and Carrigan (2008) agree that the use of contextualized problems in a Cognitively Guided Instruction classroom provide students with real-world applications of mathematical concepts. Jacobs and Ambrose (2008) classify them as powerful tools to engage students in mathematics which they enjoy to solve. One of the most powerful aspects of these problems is that students are able to model the relations and actions provided by the problems, giving them that important first step to take on their solution path. Behrend (2001) goes on to say they prevent the quick fix rule most students seek to employ, allowing them to develop the mathematical understanding the problem is designed to address. With understanding comes the ability to represent contextualized problems with symbolic mathematical notation, as noted by Jacobs and Phillip (2004). This gives students a powerful tool in their working careers, as many employers expect this ability to translate problems at the workplace into symbolic notation. Keterlin-Geller et al. (2007) state that this is especially true for the science and technology fields; including jobs such as chemical equations, computer simulation and drug dose interactions.

Contextualized problems also increase student participation and interest. Carpenter and Levi (2003) suggest this is due to the fact that children believe mathematics makes sense and they could make sense of these types of problems. She went on to state students would work on word problems for longer periods of time, because they could find a solution to them. Jacobs and Ambrose (2008) and Kaput (2000) agree, stating that students will work to make sense of such problems, construct generalizations, and derive formalizations from conceptual activities and examples. As students are working on contextualized problems, Chen (2012) states children learn and accept that struggling with a problem is a positive product of learning, and learning from their mistakes is a powerful part of their education. Jacobs and Phillip (2004) take this even further, asking students questions that allow them to reflect on previous problems, experiences and strategies in order to identify improvements is one of the CGI teacher's goal.

Students will see their classroom learning not as material handed to them by the teacher, but as learning driven by their own experiences and inquiry. Strong et al. (2004) methods of: strategy rotation, flexible grouping and personalized learning allow students the freedom and power to explore mathematical knowledge on their own. When students are able to do this, they fulfill the vision of Loef Franke and Kazemi (2001), that they will believe that they can construct new knowledge and learning through their own investigations, making them successful independent learners. According to Kloosterman and Gainey (1993), these successful independent learners are able to successfully connect formal mathematical procedures and their own mathematical skills, allowing teachers to

apply them to new situations and generate solutions that are appropriate. These are the types of skills teachers strive to develop in their students.

### **Can Cognitively Guided Instruction Enhance Algebra Instruction?**

One focus of the National Council of Teachers of Mathematics (2000), Carpenter, Fennema, Loef Franke, Levi, and Empson (2000), and Carraher et al. (2006) is that students in the middle grades should learn algebra as both conceptual sets tied to relationships as well as a thinking style to formalize patterns, functions and generalizations that they encounter. Cognitively Guided Instruction can help the classroom teacher accomplish this in a variety of different ways.

Another important concept according to multiple sources: Carpenter et al. (2005), Molina and Ambrose (2006), Knuth et al. (2005), is that students need an understanding of the equal sign. There is a huge misconception that the equal sign is a mathematical operation to young students, strengthened by the amount of practice young students have with “fill in the blank” problems. Through the use of contextualized story problems, the teacher takes away pure calculation processes and instead asks the student to think mathematically about the relations and procedures implied. Carraher et al. (2006) provide functions as a way to introduce students to the relational meaning of the equal sign. By using functions to teach arithmetic operations such as addition, subtraction, division and multiplication you present young students with multiple representations of those operations. Once students realize that there are multiple representations, they begin to think of the equal sign as comparing two different quantities rather than a symbol demanding them to “find the answer”. This concept of relational thinking is further supported by Empson and Levi (2011), who believe children as well as adults use

relational thinking intuitively to solve mathematics problems. This type of thinking and problem solving leads students to many algebraic properties and concepts; distributive, associative, commutative and identity properties just to name a few.

Contextualized problems inherently bring topics typically thought of only outside the mathematics classroom into the lesson, integrating mathematical concepts into students' everyday lives. This type of real world application of mathematics is one of the goals of many different mathematical and professional communities. Real world application allows students to explore a variety of solution paths, which Johanning (2004) believes is critical so that they can think algebraically and work with solution paths that are sensible and appropriate to them. Kaput (2008) then pushes this topic farther, suggesting that algebraic generalization is only achieved by extending the scope of the problem to a level where the class no longer looks at the problem itself, but at the patterns or relations provided by the problem. Creating problems with context allows students to connect pre-knowledge and experience to the problem, providing a multitude of viewpoints in which to examine the problem and find those patterns.

Carraher et al. (2001) have stated that algebraic reasoning may be tied to cognitive development, and that it may require a maturity beyond the scope of elementary and middle school students. This has been supported by Keterlin-Geller et al. (2007), they believe that by the time student are introduced to algebra in the middle school many of the student view it as subjective and arbitrary. Empson and Levi (2011) have outlined ways to avoid this problem; claiming that by listening to children's thinking the teacher can use it to guild the instruction and develop those concepts. By 1) posing problems for children to solve using their own strategies, 2) choosing or writing problems that elicit a

variety of valid strategies, 3) adjusting problem difficulty and number choices, 4) asking probing questions to clarify, 5) facilitating discussion of students' strategies and 6) identifying the important mathematics in children's thinking they are able to present students with problems that will allow them to connect their knowledge to the current mathematical objectives of the lesson. The challenge for the teacher is to listen to their students' strategies and allowing them to develop those ideas, without imposing hints or their own knowledge about the problem. By listening and assessing the students' knowledge teachers can appropriately adjust their instruction to meet those students' needs. Carpenter et al. (2003), support the use of differentiated instruction to develop these concepts. Students from primary grades to middle school can understand the underlying algebraic themes; teachers need to adjust the context and numbers to match their students' developmental and skill level.

### **Preparation for Cognitively Guided Instruction**

Desimone (2009) and Borko (2004) both agree that professional development and teacher learning derive from a multitude of sources: in-service workshops, structured topic-specific seminars, classroom experiences, professional learning communities, hallway discussions with colleagues, and student conferences naming just a few. Borko (2004) found that Cognitively Guided Instruction workshops provided teachers with extensive focus on instructional practices, and this training lasted for about four weeks. This meshed well with Desimone's (2009) critical features of successful professional development programs. They were: 1) content focus, 2) active learning, 3) coherence, 4) duration, and 5) collective participation. These features have to be supported by the program to be successful, and they focus on both the teacher individually and their

learning community as a whole- key factors to ensure sustained implementation in the classroom.

## Chapter 4: Conclusions

### What is Cognitively Guided Instruction?

CGI is a teaching methodology that shifts the focal point of the lesson away from the teacher and toward the students. The overarching philosophy is that students come into the classroom having a variety of experiences and skills they can apply to problems. Instead of directly instructing the students in a mathematical concept, the teacher's expect students can solve new problems using their own current skill set, no matter the size or sophistication of that toolbox.

The students will come into the classroom, take their seats, and be presented with a problem. The presentation of such a problem will and should vary, it could appear as: a problem of the day, a video clip, a story the teacher tells the class or even a situation inspired by a student discussion. Students will then brainstorm as a class. This addresses the learning style of the interpersonal students, those students who learn through conversation, association and personal experiences.

Discussions about the problem and personal experiences with the problem are shared. During this time the teacher will informally assess each student for understanding of the problem, and make sure appropriate questions are asked so that the important details of the problem come out without the teacher explicitly relating them. Teachers should allow as many different participants into the conversation as possible. Students will be able to compare their thoughts about the problem with multiple sources. They should be able to find at least one connection between a peer's approach and their own, if they do not then the teacher will need to make adjustments for the next discussion to make sure those connections are available. After the short discussion session, each

student is expected to work on the problem individually for a set period of time. This will address the learning style of the mastery students, students who work step-by-step through their own processes to solve a problem.

The teacher's job during this segment of the class period will be moving through the classroom monitoring and mentoring students as they come to a conclusion on how the problem will be solved. Students will solve the problem with whatever method they deem appropriate. This is an opportunity for teachers to individually assess each (or as many as possible during the time constraint) student on their understanding of the mathematical concepts for the lesson. An actual solution is not the focus, that students understand the problem individually and have some sort of plan of action is crucial.

Once all students have either a solution or a plan for solving the problem, the teacher then brings the classroom back together as a whole for discussion. This next discussion phase will address those students who use multiple representations and strategies to complete a problem, also known as the self-expressive learners of the classroom.

This group discussion allows the students themselves to become the focal point of learning. They present their solutions (or solution paths) to the class through whatever means the student and teacher deem appropriate. SMART Board, overhead, document cameras, whiteboard or computer displays are some possibilities for students to display their work in a manner that all of the students can view it. Presenting students are expected to explain their solution, what they were thinking while they were working on the problem, and to justify why their answer is correct. Their peers are expected to ask questions and as a class they determine whether the solution is valid. The teacher then asks the class for a different solution or approach to the problem, and the process starts

over. The goal is to allow as many different solution paths to be presented and explained as possible, and the teacher will know how many there are from the individual work time phase of the class. Whenever a student presents their solution, a means of storing or displaying their work should be used for the final stage of the CGI phase, which addresses the children with an understanding learning style. These children examine the patterns and reasoning contained within the problem.

This phase is crucial for teaching with understanding and being able to efficiently display multiple strategies will impact the success of the students. During this time, students will examine all of the solution options that were presented to the classroom. They will look for patterns, and try to find commonalities between solution paths and choices. Each student in the class will have the ability to connect their thinking to their peers', and evaluate how their peers' thinking connects to their own. The teacher should moderate the discussion to promote understanding of how each solution is connected, and how this understanding could be used to solve similar problems. Once students have a strong understanding of their own strategies, the teacher then presents them with the formal mathematical concept underlying the problem. Children can then discuss how their solutions and methods compare to the formal ones, what similarities or differences between the two occur and what steps are needed to connect both the student's concept and formal concept into a unified mathematical procedure. Strong connections between the formal concept and student approach occur, which creates a deeper understanding of the topic and allows students to effectively apply the knowledge to new problems.

Once the students have examined the problem in its entirety, students are then expected to practice their new found knowledge in multiple ways. Some suggestions are,

but not limited to: completing a problem set, applying their knowledge to similar problems, or even creating a story problem using the same mathematical concept themselves. Drill and practice is still effective after students have a strong understanding of the concept they are expected to perform. Applying their knowledge is a great assessment for how well the students understand how to utilize their new skill set to a comparable situation. Student created problems are a powerful tool; not only are you able to assess a student's knowledge about a concept but you are also able to motivate peers by using them in your classroom. Students enjoy the challenge of solving a peer's proposed problem, and sometimes these can be used as a formal assessment for the class.

### **Do Students in a CGI Classroom Develop a Strong Understanding Between Algebraic Concepts and their Real World Applications?**

The use of CGI in the classroom will promote connections and understanding of algebraic concepts. The use of CGI allows the teacher to take mathematics out of the textbook and infuse it into the student's lives. Through the use of contextualized story problems teachers can custom- tailor scenarios to each class. This will capture the students' attention and allow them to see the application of their problem versus computing an answer. Students are more apt to spend longer lengths of time on these story problems. They will try multiple strategies to find a solution, spend more time thinking of why that strategy does, or does not, work and be able to justify to the class why they chose their particular strategy and if it works. Students are solving problems individually, discussing them as a group, presenting and defending their own work, analyzing the work of peers, finding connections between multiple solution paths, and critiquing solutions to determine which are valid.

These teaching methods initiate student participation and promote learning with understanding. They are constructing their own knowledge about mathematical concepts using previous experiences and skills. The constant demand on students to recall prior information reviews and reinforces knowledge what students previously learned.

Students start with a concrete representation of a problem and transform it into algebraic knowledge and mathematical concepts. For example, students could enter the classroom one day and have a balance at their workstation. One side of the balance would have two boxes and five pebbles on it, the other one box and 12 pebbles. The task of the students is to determine how much each box weighs. They would not be able to weigh an individual box until they were asked to justify their answer. Additional examples would be explored using different figures for the box, allowing for a difference in weight. Class discussions on how each example was solved would occur. Before students left the classroom for the day, they would have to journal about what they did that day. The next day, students would be presented a number of open number sentences. One strategy students may devise is drawing diagrams for the sentences. This would open the discussion to how open sentences relate to the balanced scales. Before students left class, they would again journal about what they did that day, and what connections it had, if any, to their previous work. In the next session, the class would go back and examine their box-scale problems. They would be asked to make a general statement linking the number sentences to the box-scale problems. They would have ideas in their journals they could draw from. Students would then examine all the possible statements or rules created by the class and try to refine it to one or two. The teacher would then link the formal concept of comparing equal quantities to the student's rule. Students would

journal this relation between the formal concept and the class rule. In a later session, the teacher would extend the problem, adjusting it to be slightly different. Students would then use their prior knowledge from the box-scale problems to solve a new scenario. Different student solution paths would be presented and compared, and a new rule would be developed if needed. This type of pattern would occur for any topic that is presented in class. In this example, there is less of an emphasis on computation and more on problem solving, allowing students to produce multiple solution strategies. Students can compare their own understanding of the mathematics involved to these strategies and create connections to help them solve similar problems.

Much of the classroom dynamic is driven by student experience and learning, not direct instruction by the teacher. By shifting the focus of the class away from the teacher, an environment of high expectations for learning and understanding is created. Students are no longer memorizing curriculum, they are interacting with it, on both a personal and group level to form connections and knowledge that are not normally achieved in a traditional classroom. This creates confidence in the student learner that they can find a solution to any problem presented to them, regardless of how they start the problem. This confidence becomes practical experience they will be able to apply and translate to work-related situations. Teachers are able to sit back and informally assess students' understanding more effectively. Through communication of proposed solutions and the justifications of those solutions teachers are able to construct a clear picture of students' knowledge and adapt their instruction and curriculum to address misconceptions from occurring.

### **How Does CGI Impact the Interest Level of Students in the Classroom?**

As soon as the teacher places meaningful context into a problem, students immediately become engaged. The use of student or teacher names, school sports teams, or local area landmarks in story problems draws the attention of the students. The same is true if the teacher uses a current event from the community, a theme or unit from another class, or even a newly released movie. When elaboration of this type is used, students are less likely to try routines or methods previously learned and actively make sense of the problem instead. Problems that may appear alien and formidable become concrete and manageable to the student. Consider the following example:  $f(x) = -3X + 20$ , *find  $f(4)$* . This type of problem, without context, becomes a series of rules and operations to perform. When presented with the problem in a contextualized format: *you start the week with twenty dollars. Each day you buy lunch at school, which costs three dollars, how much money do you have after 4 days?* Students immediately understand the scenario presented and can relate to the rate of change of the problem.

CGI allows students to share their solutions and to evaluate the solutions proposed by their peers. They are expected to systematically present their proposed solution and then justify why their approach and solution to the problem are correct. They are also expected to analyze and ask questions about their peers' work. Students then evaluate all possible solutions presented and determine which ones are valid. They communicate mathematically with each other; describing their own thinking, asking questions about why a solution is correct, asking their peers for help when they are stuck on a problem. This creates personal ownership in their learning and in their classroom that results in increased student participation.

CGI also fosters resilience in problem solving efforts and methods. Students understand there are multiple ways to solve a problem so they are more likely to formulate a plan for solving the problem. They know they are likely to choose a path that will not correctly produce a solution; but are able to examine their work, determine where they made the error and correct it. Students are aware that they are building life skills in mathematics; they understand your first approach to a problem might not be the best, most efficient, or even the correct one. They see the connections between mathematics and the world around them; they believe mathematics should make sense and that they can actually find a solution.

Through the use of CGI in the classroom, the teacher is also able to provide a variety of instructional techniques. Teachers are able to change the focus and tempo of the classroom to meet the needs of the students, and in doing so they are able to address multiple learning styles. This is dictated by the individuals in each classroom, so the atmosphere between different sections will vary. Changes in grouping, presentation, physical locality and work assignments create a learning environment where students are engaged, supporting and assessing the work that is presented to them. With the assortment of opportunities available to students over the course of the class, students can not help but become interested and involved.

### **What are Key Components for Students Understanding Algebra?**

The first key component for students' understanding algebra is the expectation that elementary students will begin to adapt their mathematical thinking into algebraic thinking. Some suggested adjustments that need to occur are:

- *Increased focus on relations, not calculations.* Students who enter algebra courses will see  $4 + 6$  and want to compute the answer, 10. But this is an incorrect response to the equation  $4 + 6 = \_\_ + 17$ , which is 7.
- *Operations and their inverses.* Students will know that the problem of “the sum of 4 times a number and 7 is 23” means for them to subtract 7 from 23 and divide by 4. But in algebra classes, they will be expected to generate an expression,  $4x + 7 = 23$ , and use properties of addition and multiplication to solve.
- *Representing and solving problems, not merely answers.* As stated in the preceding bullet, there is a difference in finding a solution and mathematically representing a problem and following a set of procedural operations to find the answer.
- *Focus on letters and numbers, instead of numbers alone.* Have students work with letters that may be unknowns, variables or parameters. Compare relations and equivalence based on properties rather than numerical evaluation.
- *Redefining the equal sign to students.* Students must understand that it is a relational symbol instead of an operational one.
- *True/false sentences: they will pinpoint errors in thinking and promote relational thinking.* These are great for evaluating student knowledge on mathematical concepts. They can be used for many different types of assessment.

- *“Missing addend” problems (open sentences).* The example of  $8 + \underline{\quad} = 15$ , is a great supplement to traditional problems where the unknown quantity comes after the equal sign.
- *Connecting function tables, number lines and graphs.* Having student represent situations in multiple forms and being able to discuss the problem based on any form chosen will strengthen students’ algebraic thinking and problem solving.

This will start young students down the path to algebraic thinking and inquiry, which will not only strengthen their computational skills but give them a deeper understanding of the mathematical concepts to which they are exposed.

Connecting algebra to relevant topics or experiences in students’ lives is another important component for understanding algebra. This gives the students a practical starting point for the problem and it allows them to directly model the situation. Once students are able to create a model or visualize the problem, they create their own solution path to the problem. By constructing their own solution instead of parroting one from their teacher, they are able to communicate effectively about the correctness of their or other’s solutions. They can also then take these strategies and generalize the concept, applying their algebraic thinking to different scenarios and problems. An effective assessment for this is to have students create their own word problem that addresses the algebraic concept.

Implementing these adjustments into the classroom creates a multitude of formative assessments that can guide instruction. By monitoring the curriculum, teaching and classroom; teachers are able to make sure they promote student learning with

understanding. By changing their classroom emphasis from “getting the right answer” to “model, solve, and explain your solution”, the teacher stops students from only developing an informal, fragmented mathematical skill set. They do create an environment where students will gain the independence, confidence and proficiency to become successful learners not only in their mathematics class, but life as well.

### **Could CGI Methodologies Help Students Learn Algebra?**

Cognitively Guided Instruction creates an atmosphere of positive, productive learning in the classroom. It opens a door to young students and algebraic thinking, notation and understanding; symbols, number lines, graphs, and function tables are all tools required in their learning in later grade levels. The use of contextualized problems connects algebraic concepts to real world applications, which increases student involvement and participation. Although studies between CGI and 8<sup>th</sup> through 12<sup>th</sup> grade algebra could not be found, the author has found a connection between the methodologies and successful algebra instruction.

The important skills for listening to children’s thinking and using it to guide instruction are 1) posing problems for students to solve using their own methods, 2) choosing and writing problems that produce a variety of solution paths, 3) appropriately adjusting problem difficulty dependent on your students, 4) ask deep, thought-provoking questions to explain, clarify and extend student thinking, 5) facilitate constant group discussion of students’ strategies along with comparison and contrast to other strategies so that students can derive new mathematical connections, and 6) identifying and connecting student derived concepts to formal mathematical ones. Teachers who can implement these elements into their classroom create an atmosphere where students guide

their own learning, and feel confident they can contribute in a positive manner. Students will recognize that errors are a part of the learning process, and they will embrace the challenge to find the correct solutions. All of these skills will increase the classroom's effectiveness in teaching any mathematical concept, not solely algebra.

**What Type of Extra Resources, Preparation, or Professional Development is needed for Teachers to Implement CGI into their Classrooms?**

The most powerful asset CGI brings to the curriculum is that no additional classroom materials are needed. Teachers are able to take their current curriculum and transform the problems present in it to rich, robust story problems that will challenge students. Creating these types of story problems is by no means easy, it often helps to have another colleague with whom to collaborate ideas and details. Colleagues could include: other mathematics teachers, the core teacher with whom the teacher is collaborating, community members or elders, or local business professionals. Teachers should keep in touch with local businesses so they are kept up to date on what application skills students need to be successful in jobs, they can then integrate these skills into their classroom. The CGI story problems created should have meaning, problems students can see as important to themselves or to a real world situation they may encounter at some point in their lives. These steps will ensure the teacher is providing the students with this opportunity.

There are different types of preparation teachers will need to implement Cognitively Guided Instruction into their classroom. After they have adapted a story problem to fit their current focus of the lesson, teachers must be prepared in a number of different ways. They must first be proactive with students when presenting the problem,

success of the lesson depends on students fully understanding what the story problem is conveying for information and what end goal they need to accomplish. Teachers must also mediate the classroom presentations and discussion to make sure that valid approaches are recognized by the students. They must also lay the framework for creating connections between the student's solutions and the formal operations they are performing. Making sure students see the connection and understand how their knowledge replaces and supports the formal process is critical. Teachers also need to guide the classroom throughout the lesson, making sure everyone is participating and contributing. When all of the students feel they can productively talk about mathematics with the class, they will persevere through more challenging problems because they believe they find a solution.

As for professional development, teachers need to be trained in the appropriate use of CGI in the classroom. Teachers need to observe the use of Cognitively Guided Instruction in their subject, or they need to become the students and experience lessons presented in this style. A trainer or local instructional coach must be available to provide reference support, observations and feedback on instruction. Regular professional learning communities for mathematics teachers also need to be implemented within the district, so teachers have time to collaborate on effective ideas and techniques they use in the classroom. If a teacher plans on co-curricular lessons, extra planning time would be needed as well for those teachers involved. Just as CGI creates connections between mathematical concepts and real world applications for students, it demands connections between the teacher and their colleagues so that students are receiving the highest quality instruction possible.

### **Problems with the Research**

When the author started to research the topic of CGI implemented within an eighth grade mathematics classroom, there were a few issues that either complicated the research or did not align with the original goal. Some of these include: the age of children studied participating in a CGI classroom, the mathematical areas where CGI is implemented and direct references between effective mathematical instruction practices and CGI.

All of the CGI articles and studies the author reviewed took place in the elementary school, typically around the third grade. While all children, and even adults, display the same sequences of approach for new problems presented to them, it was unclear how the challenge of eighth grade algebra concepts would influence student motivation and achievement. Although there has been speculation about children's cognitive development and the learning of abstract concepts introduced in algebra, there also has been considerable research on introducing algebraic reasoning in younger grades. All of these studies have shown that younger students can think algebraically to various degrees and that these concepts should be introduced at the elementary level.

Since most of the CGI studies were done in the elementary classroom, the content of those lessons are based more upon mathematical concepts where a direct concrete model is available. This is not true of algebra, where many teachers expect students to replace concrete thinking with generalization- an application of a rule or function to an unknown amount. This creates a delicate balance for the teacher when designing lessons and story problems for their CGI classroom. They need to make sure they have a concrete reference for their students and that the problems are designed in such a manner

where it will lead students to question how it would apply to different values, or a general application overall.

The studies also did not seem to place value upon a diverse classroom. They seemed to assume that any student from any class will make connections and draw upon their experiences to solve the problem. Teachers need to have a good assessment of their students for each class, and be able to anticipate any concerns or questions students may have that would cause them encounter a barrier in the problem.

The last concern the author had while doing his research was tying methodology directly to CGI. When searching for CGI resources, many resources can be found, but most of those result from the author being associated with Cognitively Guided Instructional research at some point in their careers. Very little of the research or articles are directly references to CGI. While many would say that there are definite guidelines and structures for CGI, others would argue that it is all just best practices for teaching. Very little of the research directly tied their concepts or approaches to CGI, but CGI outlined their applications as techniques used and adapted by the program. Is CGI a new concept, or one that is just a collection of all the best practices that teachers already know? One colleague asked a CGI trainer; “What is CGI?” and he responded “What do you think CGI is?” Without formalizing a set template for its practices, many teachers are left unsure of its meaning, use, adaptability or effectiveness to their classroom. This needs to change if teachers are expected to infuse this methodology into their curriculum.

### **Usefulness of This Study**

For teachers new to Cognitively Guided Instruction, the following points are essential to effectively implement this teaching strategy into the classroom:

- Classifying your level of beliefs and practices as a teacher. Identifying which style level you fit into is essential in order to create rich, contextualized story problems and guide classroom procedures so that Cognitively Guided Instruction will be an effective teaching method for your students. Those levels are: Level 1: teachers who believe students need to be explicitly taught. Level 2: teachers who provide opportunities for children to solve problems using their own strategy and show the students specific methods. Level 3: Teachers who believe students do not need a strategy provided for them. And Level 4: teachers who conceptualize learning and thinking in terms of the students in their classrooms. Teachers should challenge themselves and their beliefs, slowly moving from the lower levels to the upper, which will also challenge their students and create a classroom that promotes exploration, discussion and connections.
- Create contextualized story problems for the lesson. Through these real world examples, student will be able to recall past experiences and construct new meanings for mathematical content. Requiring students to present and defend their solution methods is a key component. Children take ownership of their learning and understanding when they see a direct connection between the classroom and the world. Their participation inspired by their peers, their knowledge tested when they examine all possibilities and validate the correct one. Students gain confidence in their mathematical skills and are more apt to attempt challenging problems when they have this confidence base that is fostered and nourished in a Cognitively Guided Instructional classroom.

- Identify your student's individual learning styles. Through the use of Cognitively Guided Instruction, all of the four learning styles (mastery, understanding, interpersonal and self-expressive) are addressed and focused upon. By differentiating the instructional presentation in class, teachers allow each student an avenue to showcase their mathematical expertise through their learning style. Some suggestions on differentiating instruction are: rotating strategy focus over the course of a unit or theme to enhance and promote growth in a learning style, flexible groupings of style-alike and style-diverse to promote positive classroom discourse, and personalized learning to either support struggling students or challenging high achieving students in your classroom.
- Teachers need a lot of planning time to implement this teaching method into their classroom. Although Cognitively Guided Instruction does not require any additional materials, it does demand an understanding of how to transform the mathematical concepts into rich, vibrant story problems that capture student's attention and interest. Cognitively Guided Instruction teachers also need frequent professional learning communities so that they can co-operate in lesson designs, implementation and suggestion. Just like in their classroom where students are required to share their solutions and justify its validity to the class, teachers need the opportunity to develop story problems, discuss how the mathematics is related and what direction the classroom discussion needs to follow, share what procedures work and what does not, in order to

maintain the high level of achievement and interaction that Cognitively Guided Instructional classroom can provide.

The author found this research particularly useful for a variety of different reasons. This topic holds particular interest with the author because Cognitively Guided Instruction in mathematics has been adopted by his district. Teachers are expected to use elements of Cognitively Guided Instruction in their classrooms everyday. As stated before, Cognitively Guided Instruction is not a curriculum, but rather an instructional methodology. That is one of the strengths of Cognitively Guided Instruction, the ability to mesh with whatever curriculum the teacher currently uses. These methodologies were also used when considering a new curriculum for the school. When examining materials and resources, the author viewed each with the Cognitively Guided Instructional classroom in mind. Things that the author looked for in new curricular materials: contextualized problems; diverse problems; real-world applications; flexibility in problem sets, chapter layouts and test questions.

The author also has personal interest in Cognitively Guided Instruction. He has had minimal training in the implementation of Cognitively Guided Instruction and wanted to learn more about it. His classroom atmosphere and expectation blend well with Cognitively Guided Instruction. The author already created a “problem of the day” using context from either his personal or school life that connected to mathematical concepts. He is currently revising his lesson plans for next year in order to allow sufficient time for contextualized problems and classroom discussion. The research filled in some of the gaps he received in his Cognitively Guided Instruction training, and he is more confident in his ability to implement this methodology in his classroom. He also

plans to continue his learning and development through professional learning communities established at his school.

One limitation the author has is that his colleagues within the middle school do not share his view on the effectiveness of Cognitively Guided Instruction, and do not plan to use it in their classrooms. He plans to continue to implement the methodology within his classroom and share his learning with high school staff, which also has a favorable approach to Cognitively Guided Instruction. The author also hopes to be able to share his training, research and classroom experience at a future math conference.

### **Suggestions for Future Research**

Cognitively Guided Instruction should be examined at a middle school, high school, and even college level. There is strong research suggesting CGI is able to create strong connections between student's knowledge and mathematical concepts. Should be implemented at every level of student instruction? A study of Cognitively Guided Instruction at the middle, high and college levels should be implemented to determine its effectiveness at those levels. If it is not effective, at what point does Cognitively Guided Instruction overtaken by another instructional method? Are there certain mathematical courses or concepts that are not compatible with Cognitively Guided Instruction? These are a few of the research questions that still need to be investigated for Cognitively Guided Instruction.

In conclusion, Cognitively Guided Instruction in an eighth grade mathematics classroom, what's the point? The point is that Cognitively Guided Instruction will transform your classroom into a unique learning environment for your algebra students. I hope that this study will be beneficial to other middle and high school teachers who are

looking to implement Cognitively Guided Instruction into their curriculum. As the research has shown, the Cognitively Guided Instruction methodology does not replace your current curriculum but rather enhances the educational experience of your students. It promotes student involvement, and ties mathematical concepts to real world applications. Given that there is a great amount of preparation required by the teacher to integrate this in the classroom, the benefits from creating story problems and challenging students to become more active in the classroom and in their education easily outweigh any upfront inconvenience experienced. Cognitively Guided Instruction is not teaching students materials to pass a test, it is establishing life skills that will allow them to be more successful in their everyday lives.

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